

A Single-Vendor Single-Buyer Integrated Inventory Model for Deteriorating Items with Ramp Type Demand

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Abstract - Most of the researchers have developed integrated vendor buyer inventory system with different assumptions on demand pattern such as constant, stock-dependent, price-dependent, pricesensitive stock-dependent, ramp type demand etc. We have considered a single-vendor single-buyer integrated inventory model for deteriorating items with ramp type demand rate for both the vendor and buyer and multiple deliveries per order. Shortages are not allowed. A mathematical model incorporating the costs of both the vendor and the buyer is considered and a solution procedure is given to find the optimal solution of the model. A numerical example is given to illustrate the model. Sensitivity analysis is carried out to study the effects of changes in the parameters on optimum total cost, optimal order quantity and the cycle length.

Keywords - Inventory system, Ramp type demand, Constant deterioration, Vendor-buyer, multiple deliveries.

1. INTRODUCTION

In a supply chain inventory system, buyer purchases products from vendor. The buyer may be a regular purchaser of goods from vendor based on long term agreements. This association builds a greater commitment to quality and hence, trusts between the buyer and vendor is developed over time. Good relationship between the vendor and buyer may allow for the sharing of information, forecasts and knowledge between them. Team up work of buyer and vendor leads to considerable success in business. Hence, coordination between buyer and vendor is an important way to gain competitive advantage in supply chain management as it lowers supply chain cost. The vendor buyer integrated inventory model aims to determine the number of shipments from vendor to buyer, the shipment schedule and the size of shipment quantity in each shipment which minimizes the integrated total cost per unit time. The integration approach has been studied by many researchers with different assumptions on the demand rate and the deterioration rate. These studies have shown that if the number of deliveries is decided in cooperation of both the vendor and the buyer, then the overall integrated cost can be minimized. (Yang and Wee [22], Ouyang et al. [13], Ben-Daya and Hariga [2], Wee et al. [21]). Ouyang et al. [13] have considered two cases. In the first case, it is assumed that the lead time demand is stochastic and it follows a normal distribution. In the second case, the inventory model is solved by using the minimax distribution free approach. Shortages are allowed during the lead time and it is shown that the lead time can be reduced at an added cost. Ben-Daya and Hariga [2] assumed stochastic demand and the lead time varying linearly with the lot-size. Wee et al. [21] have developed an inventory system with limited resources and Weibull deterioration of items.

Yang and Wee [23] have shown that the total profit of the whole inventory system can be maximized under the collaboration of vendor and buyer with permissible delay in payment. They have incorporated a negotiation factor to balance the extra profit sharing between the two players according to their contribution. Hill and Omar [5] developed a production and shipment policy with unequal shipment sizes based on the assumption that the vendor's unit holding cost exceeds the buyer's unit holding cost. The quantity discount strategy offers lower unit price for larger order quantities to allure the buyer to purchase in larger quantities. Lin and Ho [11] showed how to maximize the joint total profit per unit time by optimizing the buyers order quantity, retail price and the number of lots to be delivered from the vendor to the buyer in one production run. Just-in-time (JIT) policy plays a crucial role in the present supply chain environments. The characteristic features of JIT systems include consistency in quality, small lot sizes, frequent delivery, short lead time and closer supplier ties. Many researchers have developed vendor-buyer integrated inventory models for deteriorating items under JIT multiple deliveries. (Jong and Wee [9], Chung and Wee [3]). Chung and Wee [3] assumed that the demand rate is a function of stockdependent selling rate. If a task is repeated number of times, then the time required to complete the task decreases. This is known as the learning effect. Tsai [19] developed a production and shipment system for a single-vendor single-buyer integrated deteriorating model by incorporating this idea of learning effect. A production process need not always produce good items. Sometimes it can produce a certain number of defective items too. To ensure good quality, inspection of items produced must be considered. But there may be inspection errors also. Hsu and Hsu [6] considered imperfect quality of items and inspection errors in an integrated vendor-buyer production-inventory model. In real life, for items such as newly launched electronic components, fashionable clothes, cosmetic products, automobiles etc., the demand rate increases at the beginning up to a certain time and afterwards it stabilizes and becomes constant. Such type of demand rate is termed as ramp type demand rate. Several researchers have developed inventory models with



ramp type demand rate. Jain and Kumar [7] developed an EOQ inventory model with ramp type demand rate, Weibull deterioration rate starting with shortage and shortages being completely backlogged. Skouri et al. [16] considered inventory models with ramp type demand rate, Weibull deterioration rate and partial backlogging of unsatisfied demand. Manna and Chaudhuri [12] developed an order level inventory model with ramp type demand rate and time dependent deterioration rate. They studied two models without shortages and with shortages by assuming that the time point at which the demand is stabilized occurs before the production stops. Skouri et al. [17] extended the work of Manna and Chaudhuri [12] by considering different conditions for stabilization of demand rate. Deng et al. [4] and Panda et al. [14] discussed inventory models for deteriorating items with ramp type demand rate. Jain and Kumar [8] derived an EOQ inventory model with ramp type demand rate, three parameter Weibull deterioration rate starting with shortage and shortages being completely backlogged. Skouri et al. [18] proposed an order level inventory model for deteriorating items with general ramp type demand rate under permissible delay in payments by allowing shortages with partial backlogging. Singh and Sharma [15] developed an inventory model for deteriorating items with ramp type demand rate by allowing shortages with partial backlogging. Valliathal et al. [20] studied the inflation effects and time discounting on an EOQ model with ramp type demand rate and Weibull deteriorating/ameliorating items. Karmakar et al. [10] developed an inventory model for deteriorating items with ramp type demand rate and time varying holding cost. The model has been studied under the two different replenishment policies (i) starting without shortages (ii) starting with shortages, both with partial backlogging. Recently, Aarya and Kumar [1] developed a single vendor single buyer integrated production inventory model with ramp type demand rate for vendor and constant demand rate for buyer. They have considered three parameter Weibull deterioration, variable holding cost, inflation and multiple deliveries. In this paper, we study the single-vendor single-buver integrated inventory model for deteriorating items by incorporating ramp type demand rate for both vendor and buyer. This paper is organized as follows: In section 2, assumptions and notations are provided. In section 3, a mathematical model for deteriorating items taking into account the perspectives of both the vendor and the buyer with ramp type demand rate and the multiple deliveries per order is developed. The study is focused on the shipment schedule in terms of the size and number of shipments transferred from the vendor to the buyer assuming perfect quality. A solution procedure is also provided in this section. In section 4, a numerical example is provided to validate the proposed model. Sensitivity analysis is also carried out. Conclusions are summarized in section 5.

2. ASSUMPTIONS AND NOTATIONS

The proposed model is developed using the following assumptions and notations

2.1 Assumptions

1. Demand rate is a ramp type function of time given by

$$f(t) = \begin{cases} ae^{bt}, & \text{if } t < \mu \\ ae^{b\mu}, & \text{if } t \ge \mu \end{cases}$$

- 2. where a > 0 is the initial demand rate and 0 < b < 1 is the rate at which demand rate increases
- 3. The supply chain comprises of single-vendor, single-buyer and single-item.
- 4. Shortages are not allowed.
- 5. The lead-time is zero or negligible.
- 6. The study assumes complete cooperation of vendor and buyer.
- 7. The number of vendor's shipment is an integer.
- 8. Constant deterioration rate of the item is considered.
- 9. The deteriorated units can neither be repaired nor replaced during the cycle time.
- 10.Deterioration of the product is considered only after they have been received into the inventory.
- 11.Multiple deliveries per order are considered.

2.2 Notations

 θ Deterioration rate of items

T Time length of each cycle (a decision variable)

- *lv*1(*t*1) Inventory level for vendor when t_1 is between 0 and μ
- *Iv*2(*t*2) Inventory level for vendor when t_2 is between μ and *T*
- *Tbi* Delivery cycle time for buyer in months before μ (a decision variable), i=1,2,3,...n
- T_b Delivery cycle time for buyer in months after μ
- $I_{bi}(t)$ Inventory level for buyer when t is between 0 and T_{bi}
- $I_b(t_b)$ Inventory level for buyer when t_b is between 0 and T_b
- h_v Holding cost per unit per unit time for Vendor
- *h*_b Holding cost per unit per unit time for buyer
- A_v Vendor's set up cost per cycle time
- *A*_b Ordering cost per order for buyer
- *Cdv* Unit deteriorating cost for vendor
- *Cdb* Unit deteriorating cost for buyer
- *OCV* Set up cost per unit time for vendor
- *DCV* Deterioration cost per unit time for vendor
- *HCV* Holding cost per unit time for vendor
- *TCV* Total cost per unit time for vendor
- OCB Ordering cost per unit time for buyer
- *DCB* Deterioration cost per unit time for buyer



НСВ	Inventory holding cost per unit time for
	buyer

- *TCB* Total cost per unit time for buyer
- *TC* Integrated total cost
- *n* Number of deliveries per cycle time before
- $\begin{array}{l} \mu \text{ (a decision variable), integer valued} \\ m & \text{Number of deliveries per cycle time after} \\ \end{array}$
- μ (a decision variable), integer valued
- q_i Size of *i*th shipment from the vendor to the buyer before μ , i=1,2,3,...,n
- q_b Size of equal shipments from the vendor to the buyer after μ

3. MATHEMATICAL MODEL AND SOLUTION

Our objective in this study is to determine the optimal values of the number of shipments *n* between $[0, \mu]$, the number of shipments *m* between $[\mu, T]$, the size of each shipment and the cycle length *T*. Since we consider the ramp type demand rate, the vendor's inventory system can be divided into two phases $[0, \mu]$ and $[\mu, T]$ as shown in Figure 1. The vendor's inventory system at any time t_1 during the period $[0, \mu]$ can be described by the following differential equation $dI_{v1}(t_1) + \theta I_{v1}(t_1) = \alpha e^{bt_1} = 0 \leq t_1 \leq \mu$

$$\frac{dt_1}{dt_1} + \theta I_{v1}(t_1) = -ae^{\delta t_1}, \quad 0 \le t_1 < \mu$$
(1)

with initial condition $I_{v1}(0) = I_{mv}$ The vendor's inventory system at any time t_2 during the period $[\mu, T]$ can be described by the following differential equation:

$$\frac{dI_{v2}(t_2)}{dt_2} + \theta I_{v2}(t_2) = -ae^{b\mu}, \quad \mu \le t_2 \le T$$
(2)

with boundary condition $I_{v2}(T) = 0$.



Figure 1: Inventory level for vendor

There are n deliveries each with delivery cycle time T_{bi} , i = 1,2,...,n during the period $[0, \mu]$ and m deliveries each with delivery cycle time T_b during the period $[\mu, T]$ per order from the vendor to the buyer (Figure 2). The buyer's inventory system at any time t during the

 i^{th} replenishment cycle $[0, T_{bi}]$ can be described by the following differential equation:

$$\frac{dI_{bi}(t)}{dt} + \theta I_{bi}(t) = -ae^{bt}, \quad 0 \le t \le T_{bi}$$
(3)

with boundary condition $I_{bi}(T_{bi}) = 0$ where i=1,2,3,...n and $T_{b1} + T_{b2} + T_{b3} + ... + T_{bn} = \mu$ The buyer's inventory system at any time t_b during $[0, T_b]$ can be described by the following differential equation:



Figure 2: Inventory level for buyer

with boundary condition $I_b(T_b) = 0$ and $mT_b = T - \mu$ the solutions of these differential equations are

$$I_{v1}(t_1) = I_{mv}e^{-\theta t_1} - \frac{a}{(b+\theta)}(e^{bt_1} - e^{-\theta t_1}), \quad 0 \le t_1 < \mu$$
 (5)

$$I_{v2}(t_2) = \frac{ae^{b\mu}}{\theta}(e^{\theta(T-t_2)} - 1), \quad \mu \le t_2 \le T$$

(6)

$$I_{bi}(t) = \frac{a}{(b+\theta)} (e^{(b+\theta)T_{bi}-\theta t} - e^{bt}), \quad 0 \le t \le T_{bi}$$
⁽⁷⁾

$$I_b(t_b) = \frac{ae^{b\mu}}{\theta} (e^{\theta(T_b - t_b)} - 1), \quad 0 \le t_b \le T_b$$
(8)

From figure 1, $I_{v1}(\mu) = I_{v2}(\mu) = Q_v$; hence from (5) $Q_v = I_{mv}e^{-\theta\mu} - \frac{a}{(b+\theta)}(e^{b\mu} - e^{-\theta\mu})$ (9)

and from (6)

$$Q_{v} = \frac{ae^{\beta\mu}}{\theta} \left(e^{\left(\theta \left(T-\mu\right)\right)} - 1 \right)$$
(10)

From (9) and (10)

$$I_{mv} = Q_v e^{\theta \mu} + \frac{a}{(b+\theta)} (e^{(b+\theta)\mu} - 1)$$
 (11)

From figure 2, $I_{bi}(0) = q_i$; hence from (7)

$$q_i = \frac{a}{(b+\theta)} \left(e^{(b+\theta)T_{bi}} - 1 \right) \tag{12}$$

and
$$I_b(0) = q_b$$
; from (8)

$$q_b = \frac{ae^{b\mu}}{\theta} (e^{\theta T_b} - 1)$$
(13)

During the cycle time *T*, there are *n* replenishments in $[0, \mu]$ and *m* replenishments in $[\mu, T]$ to the buyer.



The buyer's ordering cost is

$$OCB = \frac{(n+m)A_b}{T}$$
(14)

The buyer's holding cost is

$$HCB = \frac{h_b}{T} \left[\sum_{i=1}^n \int_0^{T_{bi}} I_{bi}(t) \ dt + m \int_0^{T_b} I_b(t_b) \ dt_b \right]$$
(15)

The buyer's deteriorating cost is

$$DCB = \frac{C_{db}}{T} \left[\sum_{i=1}^{n} \left(q_i - \int_0^{T_{bi}} f(t) \, dt \right) + m(q_b - f(\mu)T_b) \right]$$

= $\frac{C_{db}}{T} \left[\sum_{i=1}^{n} \left(q_i - \int_0^{T_{bi}} ae^{bt} \, dt \right) + m(q_b - ae^{b\mu}T_b) \right]$ (16)

Therefore the total cost for the buyer is

$$TCB = OCB + HCB + DCB$$
(17)

The vendor's set up cost is

$$OCV = \frac{A_v}{T}$$
 (18)

The vendor's holding cost is

$$HCV = \frac{h_{\pi}}{T} \left[\int_{0}^{\mu} I_{e1}(t_1) dt_1 + \int_{\mu}^{T} I_{e2}(t_2) dt_2 - \left(\sum_{i=1}^{n} \int_{0}^{T_{bi}} I_{bi}(t) dt + m \int_{0}^{T_{b}} I_{b}(t_b) dt_b \right) \right]$$
(19)

The vendor's deteriorating cost is

$$DCV = \frac{c_{dv}}{T} [I_{mv} - (\sum_{i=1}^{n} q_i + mq_b)]$$
(20)

Since shortages are not allowed, $I_{mv} - (\sum_{i=1}^{n} q_i + mq_b)$ cannot be negative. To ensure this, we include the constraint $I_{mv} \ge \sum_{i=1}^{n} q_i + mq_b$ Therefore the total cost for the vendor is TCV = OCV + HCV + DCV (21)

The integrated total cost for the vendor and the buyer is TC = TCB + TCV (22)

3.1 Solution Procedure

The objective of the problem is to determine the optimal values of the number of shipments n and m, the shipment schedule T_{bi} where i=1, 2... n and the cycle length T that minimize TC. Since the number of shipments n and m are Discrete variables, we can determine the value of n and m by the following procedure.

Step 1: For a choice of (n, m), n=2, 3... and m=1,2,3,..., consider the partial derivatives of TC with respect to $\frac{T_{bi}}{\partial TC}$ and T and equate to zero. $\frac{\partial TC}{\partial T_{bi}} = 0$ where i = 1, 2, ..., n and $\frac{\partial TC}{\partial T} = 0$

Solve these simultaneous equations and hence find T_{bi} where i=1, 2... n and *T*, and then calculate *TC* by equation (22).

Step 2: The optimal values of *n*,*m*, *_{Tbi}* where i=1,2,...,n and *T* denoted by n^*,m^*,T_{bi}^* and T^* are obtained such that

Step 3: Find the shipment size q_i where i=1, 2... n from equation (12), q_b from equation (13), Q_v from equation (10) and I_{mv} from equation (11). Since the integrated total cost function given in equation (22) is a function of n+3 variables, we solve for the optimal values of n^*, m^*, T_{bi}^* where i=1,2,...,n and T^* by using pattern search procedure in MATLAB.

4. NUMERICAL EXAMPLE AND SENSITIVITY ANALYSIS

In order to illustrate our proposed model, we consider the following numerical example. A_v =\$600, A_b =\$100, $h_v = \$0.9, h_b = \$1.1, C_{dv} = \$2.0, C_{db} = \$2.5, a = 100, b = 0.08,$ θ =0.1, μ =0.12. We obtain the optimal solution as follows: $n^*=2$, $m^*=1$, $T^*=3.259$, $T_b^*=3.14$, $q_b^*=372.31$, *TC**=\$521.03, *T*_{bi}*=0.06 for i=1,2 and *q*_i*=6.03 for i=1,2. I_{mv} =388.94, Q_v =372.31. From the table 1, it is observed that the total cost for the system (TC) is \$521.03 (TCB = \$321.62 and *TCV* = \$199.42) from the integrated perspective and it is \$545.63 (*TCB* = \$216.13 and *TCV* = \$329.49) from the buyer's perspective. It is also observed that as the value of *n* increases (n = 2, 3, 4, ...), the vendor's total cost (TCV) decreases. But, both the integrated total cost (TC) and the buyer's total cost (TCB) increases. Thus, if the decision is made solely from one perspective it will cost the other. The numerical example results indicate that the integrated policy gives impressively lower total cost compared to any independent decision taken from either the vendor or the buyer's perspective.

Table 1: Optimal solutions of number of deliveries (n, m)

iii j							
(n,m)	Ты, i= 1,2,п	T	тсв	тсч	тс		
(2,1)	0.06	3.259	321.62	199.42	521.03°		
(2,2)	0.06	3.584	232.71	293.84	526.55		
(2,3)	0.06	3.795	216.13 ⁵	329.49	545.63		
(2,4)	0.06	3.972	216.92	350.32	567.24		
(2,5)	0.06	4.132	224.07	365.29	589.36		
(3,1)	0.04	3.417	359.89	191.09	550.98		
(3,2)	0.04	3.74	260.71	293.14	553.85		
(3,3)	0.04	3.945	240.12	331.34	571.45		
(3,4)	0.04	4.115	238.52	353.44	591.96		
(3,5)	0.04	4.268	244.02	369.15	613.17		
(4,1)	0.03	3.565	395.66	183.96	579.57		
(4,2)	0.03	3.888	287.01	293.05	580.01		

^{*a*}The optimal solution from the integrated perspective. ^{*b*}The optimal solution from the buyer's perspective.



The current study uses the percentage of integrated total cost difference to perform sensitivity analysis of the proposed model. The percentage of integrated total cost difference is defined as:

$$PICD = \frac{(TC - TC^*)}{TC^*}$$

When one subset of the parameters set $S=\{(A_v, A_b), (h_v, h_b), (C_{dv}, C_{db})\}$ decreases by -10%, -20% or increases by +10%, +20%, the relationships between the known parameters, the decision variables and the percentage of integrated total cost difference are given in Tables (2-4) and Figure 3.



Figure 3: PICD vs. change rate of various parameters

Table 2: Sensitivity analysis when A_v and A_b are changed

A_{ν}	480	540	{600}	660	720
Ab	80	90	{100}	110	120
(n,m)	(2,1)	(2,1)	(2,1)	(2,1)	(2,1)
Τ	2.946	3.108	3.259	3.401	3.536
Tbt (i=1,2)	0.06	0.06	0.06	0.06	0.06
q ₁ (i=1,2)	6.03	6.03	6.03	6.03	6.03
ТС	462.97	492.70	521.03*	547.99	573.94
PICD (%)	-11.14	-5.44	0.00	4.92	10.15

*: the optimal total cost; {}: the base column

Table 3: Sensitivity analysis when hv and hb are changed

0						
h_{v}	0.72	0.81	{0.9}	0.99	1.08	
hb	0.88	0.99	{1.1}	1.21	1.32	
(n,m)	(2,1)	(2,1)	(2,1)	(2,1)	(2,1)	
Τ	3.529	3.386	3.259	3.146	3.044	
T_{bt} (i=1,2)	0.06	0.06	0.06	0.06	0.06	
q_i (i=1,2)	6.03	6.03	6.03	6.03	6.03	
ТС	479.25	500.59	521.03*	540.53	559.35	
PICD (%)	-8.02	-3.92	0.00	3.74	7.35	

*: the optimal total cost; {}: the base column

The main observations drawn from the sensitivity analysis are as follows:

- 1) The numerical example indicates that the integrated approach has reduced the total cost for the vendor and the buyer [Table 1].
- 2) The value of (*n*,*m*), *T*_{bi} and *q*_i where i=1,2 are not sensitive to all parameters [Table 2 to 4].

3) The graph in Figure 3, shows the percentage change in integrated total cost for percentage changes in the values of the different parameters. The percentage change in integrated total cost increases for changes in all parameters.

Table 4: Sensitivity analysis when Cdv and Cdb are changed

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Cdv	1.6	1.8	{2.0}	2.2	2.4	
Cdb	2.00	2.25	{2.5}	2.75	3.00	
(<i>n</i> , <i>m</i>)	(2,1)	(2,1)	(2,1)	(2,1)	(2,1)	
Τ	3.315	3.287	3.259	3.232	3.206	
T _{bi} (i=1,2)	0.06	0.06	0.06	0.06	0.06	
qi (i=1,2)	6.03	6.03	6.03	6.03	6.03	
ТС	511.83	516.42	521.03*	525.47	529.93	
PICD (%)	-1.77	-0.88	0.00	0.85	1.71	

*: the optimal total cost; {}: the base column

5. CONCLUSION

In this paper, we have formulated an integrated vendor- buyer inventory model for deteriorating items with ramp type demand rate. We have proposed a solution procedure to determine the optimal values of the model. The results of numerical example indicate that the cooperation of both the vendor and the buyer has reduced integrated total cost.

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