Unsteady MHD Free Convection Flow Past a Semi-Infinite Moving Vertical Plate with SORET and DUFOUR Effects

J. Venkata Madhu[1], M. N. Raja Shekar[2], B. Shashidar Reddy[1*]
[1]Department of Mathematics, SNIST, Hyderabad-501301, INDIA.
[2]Department of Mathematics, JNTUH CEJ, Karimnagar, INDIA.
*bsreddy_shashi@yahoo.com

Abstract:- This study is to examine the flow of unsteady MHD free convective fluid past a semi-infinite moving vertical plate in a porous medium considering Dufour and Soret effects. The consequential governing equations are transformed into non linear ordinary differential equations using similarity transformations. By using Quasi-linearization technique non linear momentum equation is linearized and then the set of coupled linear differential equations are solved numerically by using implicit finite difference scheme. Then the system of algebraic equations is solved by using Gauss-Seidal iterative method. The numerical solution is dependent on different parameters such as Magnetic field parameter M, Suction parameter v_0, Soret number Sr, Dufour number D, and Darcian parameter Da. Arithmetical consequences are tabulated for the local Nusselt number and Sherwood number. Velocity, Temperature and Concentration profiles drawn for dissimilar controlling parameters disclose the tendency of the solution.

***Keywords*:-** Magnetic field parameter, Finite Difference Scheme, Soret and Dufour effects, Nusselt number.

1. INTRODUCTION

Combined heat and mass transfer in fluid-saturated porous media finds applications in a mixture of science and engineering processes such as petroleum reservoirs, geothermal and geophysical engineering moisture migration in a fibrous insulation and nuclear waste disposal and others. Twofold diffusive flow is driven by buoyancy due to temperature and concentration gradients. Bejan and Khair [1] (Bejan and Khair 1985) investigated the free convection boundary layer flow in a porous medium due to combined heat and mass transfer.

MHD free convection flows have various applications in the different fields like magnetospheres, aeronautical plasma flows and electronics. Raptis [2] (Raptis 1986) deliberate scientifically the case of time changeable two dimensional natural convective flow of an incompressible, electrically conducting fluid along an infinite vertical porous plate embedded in a porous medium. Elabashbeshy [3] (Elabashbeshy 1997) studied heat and mass transfer along a vertical plate in the presence of magnetic field. Chamkha and Khaled [4] (Chamkha and Khaled 2001) investigated the problem of coupled heat and mass transfer by magneto hydrodynamic free convection from an inclined plate in the presence of internal heat generation or absorption.

In the combined heat and mass transfer processes, concentration gradients are referred to as the dufour effect and the soret effect is the contribution to the mass fluxes due to temperature gradients. Kafoussias and Williams [5] examined the dufour and soret effects on mixed free-forced convective heat and mass transfer along a vertical surface, various other influences that have been considered include magnetic field [6], variable suction [7] and chemical reaction [8].

A range of new aspects dealing with the soret effect on the combined heat and mass transfer problems have been also studied. For example, Joly et al [9] used the Brinkman-extended Darcy model to examine the effect of the soret effect on the onset of convective instability. Mojtabi et al[10] studied, stability analysis of the influence of vibration on soret-driven convection in porous media using pseudo-spectral chebyshev collocation method. Bourich et al[11] carried out an analytical and numerical study of the onset of soret convection in a horizontal porous layer subjected to a uniform vertical magnetic field.

Recently Alam et al [12] has studied Dufour and Soret effects on unsteady MHD free convection and Mass transfer flow past a vertical porous plate in a porous medium. The current work aims to study the effects of Dufour and Soret on unsteady free convection and mass transfer flow past an infinite vertical porous flat plate in porous medium.

2. MATHEMATICAL ANALYSIS

An unsteady two-dimensional flow of an incompressible and electrically conducting viscous fluid, along an infinite vertical porous flat plate embedded in a porous medium is considered. X-axis is assumed to be along the infinite plate in the direction of the free stream which is vertical and the y-axis is taken normal to the plate. A magnetic field B_0 of uniform strength is applied transversely to the direction of the flow.

Let the temperature of the fluid and the plate is T_∞ initially with C_∞ as concentration level at all points. For $t>0$, the plate starts moving impulsively in its own plane with a velocity U_0, its temperature is raised to T_ω and the concentration level at the plate is raised to C_ω. Except the variations in temperature and concentration, it is assumed that fluid has constant properties.

Under the above assumptions, the physical variables are functions of y and t only. The equations governing the flow with the Boussinesq and boundary-layer approximation and using the Darcy-Forchheimer model, are given by

$$\frac{\partial v}{\partial y} = 0 \quad ...(1)$$

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) + g\beta^*(C - C_\infty) - \frac{\sigma\beta_0^2 u}{\rho} - \frac{\nu}{k}u - \frac{b}{k}u^2 \quad ...(2)$$

$$\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial y} = \propto\frac{\partial^2 T}{\partial y^2} + \frac{D_m K_T}{c_s c_p}\frac{\partial^2 C}{\partial y^2} \quad ...(3)$$

$$\frac{\partial C}{\partial t} + v\frac{\partial C}{\partial y} = D_M\frac{\partial^2 C}{\partial y^2} + \frac{D_m K_T}{T_m}\frac{\partial^2 T}{\partial y^2} \quad ...(4)$$

Initially (t=0) the fluid and the plate are at rest. Thus the no slip boundary conditions at the surface of the plate for the above problem for t >0 are:

$$u = U_0, v = v(t), T = T_w, C = C_w \; at \; y = 0 \quad ...(5a)$$
$$u = 0, T = T_\infty, C = C_\infty \; as \; y \to \infty \quad ...(5b)$$

Here u, v are the Darcian velocity components in the x-and y-directions respectively, t is the time, **ν** is the kinematic viscosity, g is the acceleration due to gravity, ρ is the density, β is the coefficient of volume expansion, β^* is the volumetric coefficient of expansion with concentration, k is Darcy permeability, b is the empirical constant, B_0 is magnetic strength, T and T_∞ are the temperature of the fluid inside the thermal boundary layer and the fluid temperature in the free stream, respectively, while C and C_∞ are the corresponding concentrations. Also σ is the electric conductivity, α is the thermal diffusivity, D_m is the coefficient of mass diffusivity, c_p is the specific heat at constant pressure, T_m is the mean fluid temperature, K_T is the thermal diffusion ratio and c_s is the concentration susceptibility.

3. METHOD OF SOLUTION

Now in order to obtain a local similarity solution (in time) of the problem under consideration, we introduce a time dependent length scale δ as

$$\delta = \delta(t) \quad ... (6)$$

In terms of this length scale, a convenient solution of the equation (1) is considered to be in the following form:

$$V = v(t) = -v_0\frac{v}{\delta} \quad ...(7)$$

here $v_0 > 0$ is the suction parameter.

We now introduce the following dimensionless variables:

$$\eta = \frac{y}{\delta}, = u_0 f(\eta), \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \; (\eta) = \frac{C - C_\infty}{C_w - C_\infty} \quad ...(8)$$

Then introducing the relations (6)-(8) into the equations (2)-(3) respectively, we obtain (by using the analysis of Sattar and Hossain [9], see also Hasimoto[10]), the following dimensionless ordinary differential equations:

$$f'' + (2\eta + v_0)f' + G_r\theta + G_m\varphi - Mf - \frac{1}{D_a}f - \frac{R_e F_s}{D_a}f^2 = 0 \quad ...(9)$$

$$\theta'' + p_r(2\eta + v_0)\theta' + p_r D\varphi'' = 0 \quad ...(10)$$
$$\varphi'' + S_c(2\eta + v_0)\varphi' + S_c S_r\theta'' = 0 \quad ...(11)$$

Where primes denotes differentiation with respect to η and the dimensionless quantities are given by

$D_a = \frac{k}{\delta^2}$ is the local Darcy number,

$F_s = \frac{b}{\delta}$ is the local Forchheimer number,

$R_e = \frac{U_0\delta}{\nu}$ is the local Reynolds number,

$p_r = \frac{\nu}{\propto}$ is the Prandtl number,

$S_c = \frac{\nu}{D_m}$ is the Schmidt number,

$M = \frac{\sigma\beta_0^2\delta^2}{\rho\nu}$ is the Magnetic field parameter,

$S_r = \frac{D_m K_T(T_w - T_\infty)}{T_m \nu(C_w - C_\infty)}$ is the Soret number,

$D = \frac{D_m K_T (C_w - C_\infty)}{c_s c_p \nu (T_w - T_\infty)}$ is the Dufour number,

$G_r = \frac{g\beta(T_w - T_\infty)\delta^2}{\nu U_0}$ is the local Grashof number

$G_m = \frac{g\beta^*(C_w - C_\infty)\delta^2}{\nu U_0}$ is the modified local Grashof number

The corresponding boundary conditions for $t > 0$ are obtained as

$$f=1,\ \theta=1,\ \varphi=1 \ \text{at} \ \eta=0 \quad \ldots(12a)$$

$$f=0,\ \theta=0,\ \varphi=0 \ \text{as} \ \eta\to\infty \quad \ldots(12b)$$

4. NUMERICAL PROCEDURE

The equations (9)-(11) are locally similar in time but not explicitly time dependent. To solve the system of transformed governing equations (9) & (11) with the boundary conditions (12), we first linearized equation (9) by using Quasi linearization technique [13]. Then by using implicit finite difference scheme, these equations are transformed to matrix equation form.

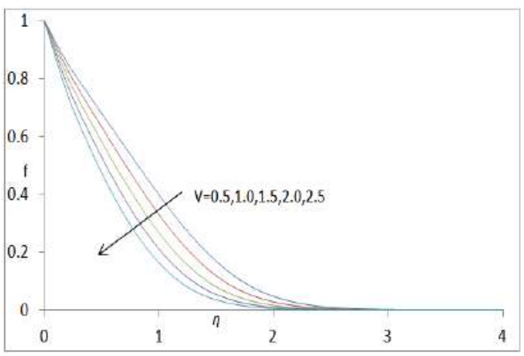

Figure1 Velocity Profile for different values of v for Sr = 2.0, D = 0.03,Da =0.5 and M =0.3

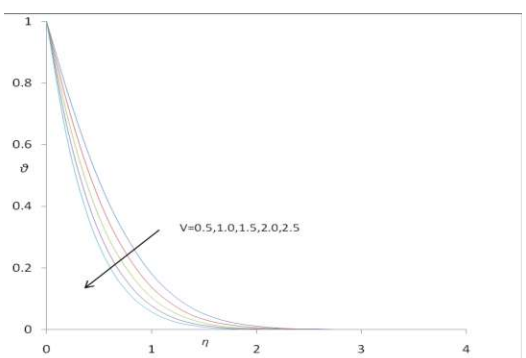

Fig2. Temperature Profile for different values of v for Sr = 2.0, D = 0.03,Da =0.5 and M =0.3

Now the computation procedure is employed to obtain the numerical solutions in which first the momentum equation is solved to obtain the values of f using which the solution of coupled energy equation is solved under the given boundary conditions using Thomas algorithm for various parameters entering into the problem and computations were carried out by using C programming.

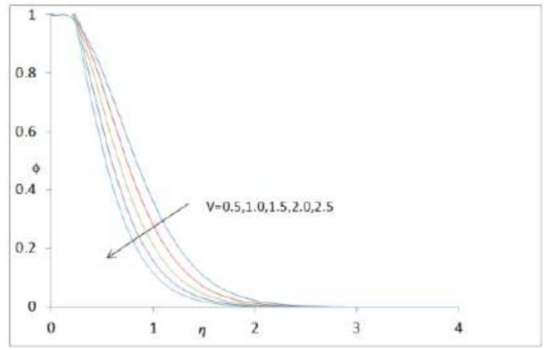

Figure 3 Concentration Profile for different values of v for Sr = 2.0, D = 0.03, Da =0.5 and M =0.3

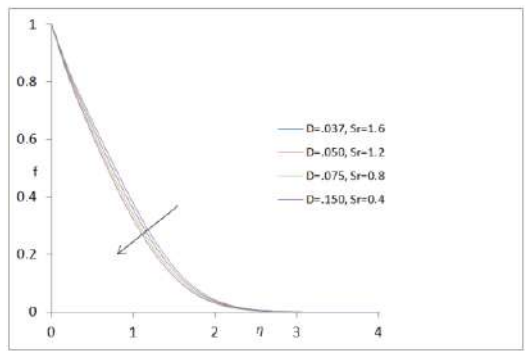

Figure 4 Velocity Profile for different values of D, Sr for v = 0.5, Da =0.5 and M =0.3

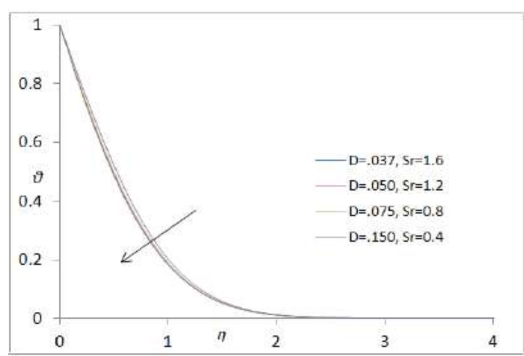

Fig 5 Temperature Profile for different values of D, Sr for v = 0.5, Da =0.5 and M =0.3

The numerical solutions of f are considered as (n+1)th order iterative solutions and F are the nth order iterative solutions. After each cycle of iteration the convergence check is performed, and the process is terminated when $|F - f| < 10^{-4}$.

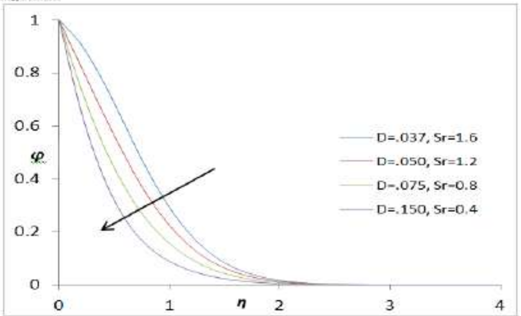

Figure6 Concentration Profile for different values of D, Sr for v = 0.5, Da =0.5 and M =0.3

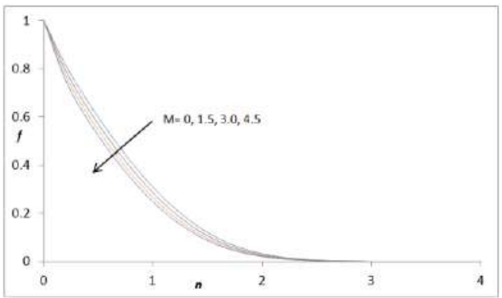

Figure 7 Velocity Profile for different values of M for v = 0.5, D =0.03 , Da = 0.5 and Sr =2.0

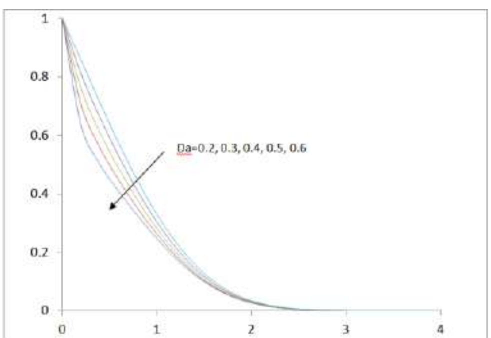

Figure 8 Velocity Profile for different values of Da for v = 0.5, M=0.3,D =0.03 , Da = 0.5 and Sr =2.0

5. RESULTS AND DISCUSSIONS

A Parametric study is performed to explore the effect of suction parameter on the velocity and the influence of Soret Dufour numbers on temperature and concentration profiles. The numerical computations have been done for different value of V_0, S_r, D and for fixed values of P_r, S_c, G_r, G_m, R_e and F_s. The values of S_r and D are taken in such a way that their product should be a constant provided the mean temperature is also constant. The value of prandtl number is chosen as 0.71 which corresponds to air and the value of Schmidt number is chosen to represent hydrogen at 25^0c and 1 atm. The values of Grashof numbers are taken to represent the free convection problem as G_r = 12 and G_m= 6. The value of local Reynolds number is taken as R_e = 100 and local forchheimer number is chosen as F_s = 0.09. The outcome of Suction parameter on the velocity profiles in shown in figure 1. It is noticed from the figure that increases in the Suction parameter decreases the velocity which indicates that Suction stabilizes the growth in the boundary layer. Figure 2 shows the effect of Suction parameter on the temperature profiles, from which it is evident that the temperature decreases as there is a rise in Suction parameter. The influence of Suction parameter on concentration profiles is displayed in figure 3. It is seen from the figure that the concentration decelerates with an increase in Suction parameter away from the wall where as a reverse phenomenon is seen near the wall, i.e. the concentration rises with the increase in Suction parameter very near to the wall. Hence the effect of Suction is to reduce the growth of the thermal and concentration boundary layers. The influence of Soret and Dufour numbers on velocity profiles and temperature profiles are shown in figure 4 and figure 5 respectively. As there is a decrease in the Soret number or an increase in the Dufour number, the velocity and temperature decelerates. Here the variation in the profiles is very small. The variation in concentration profiles with the change in Soret and Dufour number is displayed in figure 6. The concentration increases as there is an increase in Dufour number or decrease in Soret number. The effect of magnetic field on the velocity profiles is shown in figure 7. It is evident from the figure that the increase in the magnetic field parameter decreases the velocity profiles. The effect of magnetic field decreases the velocity. The variation in velocity profiles with the change in Darcy parameter is shown in figure 8. It is noticed from the figure that the velocity increases with an increase in Darcy parameter.

REFERENCES

[1]. Bejan, A., Khair, K.R.(1985). "Heat and mass transfer by natural convection in a porous medium", *Int. j. Heat Mass Transfer,* vol. 28, pp. 909-918.

[2]. Raptis, A. (1986). "Flow through a porous medium in the presence of magnetic field", *Int. J. Energy Res.*, vol.10, pp. 97-101.

[3]. Elabashbeshy, E.M.A. (1997). "Heat and mass transfer along a vertical plate with variable temperature and concentration in the presence of magnetic field", *Int. J. Eng. Sci.*, vol.34, pp. 515-522.

[4]. Chamkha, A.J., Khaled, A.R.A. (2001). "Similarity solutions for hydro magnetic simultaneous heat and mass transfer by natural convection froman inclined plate with internal heat generation or absorption", *Heat Mass Transfer,* vol.37, pp.117-123.

[5]. Kafoussias, N. G., Williams, E.W. (1995). "Thermal-diffusion and diffusion-thermo effects on mixed free-forced convective and mass transfer boundary layer flow with temperature dependent viscosity", *International Journal of Engineering Science*, vol. 33(9), pp.1369-1384.

[6]. Postelnicu, A (2004). "Influence of a magnetic field on heat and mass transfer by natural convection from vertical surfaces in porous media considering Soret and Dofour effects", *International journal of Heat and Mass Transfer*, vol. 47, pp.1467-1472.

[7]. Alam, M.S., Rahman, M.M. (2006) "Dofour and Soret effects on mixed convection flow past a vertical porous flat plate with variable suction", *Nonlinear Analysis: Modeling and control*, vol. 11(1), pp. 3-12.

[8]. Postelnicu, A. (2007). "Influence of chemical reaction on heat and mass transfer by natural convection from vertical surfaces in porous media considering Soret and Dufour effects", *Heat Mass Transfer*, vol. 43, pp. 595602.

[9]. Joly, F., Vasseur, P., Labrosse, G. (2001). "Soret instability in a vertical Brinkman porous enclosure", *Numerical Heat Transfer Part A: Applications,* vol. 39 (4), pp. 339-359.

[10]. Mojtabi, M.C.C., Razi, Y. P., Maliwan, K., Mojtabi, A. (2004). "Influence of Soret-driven convection in porous media", *Numerical Heat Transfer. Part A: Applications*, vol. 46(10), pp.981-993.

[11]. Bourich, M., Hasnaoui, M., Amahmid, A., Er-Raki, M., Mamou, M. (2008). "Analytical and numerical study of combined effects of a magnetic field and an external shear stress on Soret convection in a horizontal porous enclosure", *Heat Transfer. Part A: Applications*, vol. 54(11), pp.1042-1060.

[12]. Alam, M.S., Rahman, M.M., Samad, M.A. (2006). "Dufour and Soret effects on unsteady MHD free convection and mass transfer flow past a vertical porous plate in a porous medium", *Nonlinear Analysis modeling and control*, vol. 11(3), pp.217-226.

[13]. Bellman RE, Kalaba RE. Quasi-Linearization and Non-linear boundary value problems. New York: Elsevier;1965.