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Mathematical Modeling on Decision Based Relationship In Social Networks

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Abstract **- Srivastav et al [1]-[3] already made critical and extensive studies on mathematical modeling of social area networks. Decision theory provides a platform for nodes of Social networks to choose alternative paths of action to establish the relationship among the nodes. Decision theory has a big role on the nodes for establishing the relation among nodes. A relationship in nodes depends on the nature of the decision taken by the nodes for establishing the connectivity among the nodes. The communication among nodes depend on the decision taken by the members of the social networks. In the present work the authors have made an effort to show that the relationship among nodes of Social Networks is essentially decision based. The authors also tried to correlate the Components of Decision theory in social networks.**

Keywords **– communication, Decision theory, nodes, Social networks;**

1. INTRODUCTION

A decision is a choice made between alternative courses of action in a situation of uncertainty [17]. In the modern Social networking world, decision making is the center for all nodes of social networks. The process of establishing the different types of relation between two nodes is fundamentally a process of decision making. A Decision analysis is concerned with choosing the best act from a set of possible act, when uncertainty prevails as to the events that will occur. Decision based approaches are used either to increase the connectivity or no connectivity of nodes in social networks. Also decision based approach is used at each stage in expansion/contraction of connectivity of nodes of social networks. Passing of message or sharing of message from one node to another nodes depends on the decision making approach. In the social networks most of the activities depends on the decision making approach.

It focuses on only some aspects of activity of nodes of social networks. In particular, it focuses on how nodes of social networks can use their freedom. Nodes of Social networks do not decide continuously [18]. In the social networks there are periods/opportunity for nodes of social networks in which most of the decision-making is made, and other periods in which most of the implementation takes place. Decision-theory tries to throw light, in various ways, on the former type of period. Nodes of social networks take the choices in a non-random way. Decision theory deals with methods for determining the optimal course of action when a number of alternatives are available and their consequences cannot be forecast with certainty.

2. MATHEMATICAL DESCRIPTION OF CONNECTIVITY OF NODES IN SOCIAL NETWORK

2.1 Social network [1,2]

A social network is a social structure made up of individuals (or organizations) called "nodes", which are tied (connected) by one or more specific types of interdependency, such as friendship, kinship, common interest, financial exchange, likings or disliking, or relationships of beliefs, knowledge or prestige. Social network analysis views social relationships in terms of network theory consisting of nodes and ties (also called edges, links, or connections). Nodes are the individual actors within the networks, and ties are the relationships between the actors. Hence, mathematically, social networking can be defined as the collection of socially connected elements/objects.

i.e., set S= {social elements : social elements are connected}

In a social network S, consider two nodes (i) node1 represented by N_1 and (ii) node2 represented by N_2 and consider a function f defined between node N_1 and node N_2

$$\text{i.e., } f: N_1 \rightarrow N_2$$

such that message send by node N_1 is received by node N_2

i.e., if $m_1,m_2,m_3\ldots\ldots m_n$ be the message send by the node N_1 to N_2,then $f(m_1)$, $f(m_2)$,$f(m_3)$,……..$f(m_n)$ will be message received by N_2

i.e.,$\{m_1,m_2,m_3\ldots\ldots,.m_n\}\rightarrow\{f(m_1),f(m_2),\ldots\ldots..f(m_n)\}$

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i.e., set of send message by node N_1 → set of received message by node N_2.

2.1.1 Definition1 [3]

A social network is modeled as a graph G(V, E), where V represents a set of users embedded in a social context, and the edge set E= {(x, y)| x, y ∈V} represents friendship among users. An edge e = (x, y) is added to E when a friend request from x to y or from y to x is accepted. In SNSs (social networking sites), such as Face book and LinkedIn, edges are usually undirected. For each user u (u ∈V), the set F(u)={ x | x ∈V, (x,u)∈E}represents the friend list of u. Note, for each edge e = (x, y), x ∈ F(y) and y ∈F(x).

The mutual friends between two users can be defined as follows:

2.1.2 Definition 2 [3][4]

Given two users x and y (x, y ∈V), we define the set of the mutual friends, MF(x, y), between them as

MF(x, y) = F(x) ∩ F(y). Here, MF(x, y) stands for mutual friendship between x and y. Intuitively, the definition of the mutual friend has two properties:

Given x, y, z ∈ V, y ∈ MF(x, z) ⇔ y ∈ F(x) or y ∈F(z).

Given x, z, MF(x, z) = MF(z, x); i.e., mutual friendship is irreflexive and symmetric.

3. MATHEMATICAL DESCRIPTION IN ESTABLISHMENT IF DECISION BASED RELATION IN SOCIAL NETWORKS

3.1 Definition[12-15]

3.1.1 Connected space

Let X be a topological space. A separation of X is a pair U, V of disjoint nonempty open subset of X whose union is X. The space X is said to be connected if there does not exist a separation of X.

3.1.2 Separated Set

Let (X, τ) be a topological space. Two non empty subset A and B of X are said to be separated if and only if $A \cap B^c = \emptyset$ and $A^C \cap B = \emptyset$ .Two sets are separated if and only if they are disjoint and neither of them contains limit point of the other. Also, Subset of separated sets is separated.

3.1.3 Disconnected space

A subset E of X is said to be disconnected if and only if there exist two non-empty separated set A and B such that $E = A \cup B$.

3.2 Establishment of Direct relation between two nodes by using decision taken from each node (for connected set) in the Social Networks [1]

Let, nodeN_1 and N_2 are members of a social networks

Step1: Node N_1 takes decision to send friends request to node N_2. Here node N_1 has taken independent decision to sending the friend request to node N_2

Step2: Now node N_2 has two options either accept friend request or reject friend request. Node N_2 will take the decision to accept or reject friend request by viewing features of nodes N_1.

If node N_2 accept friend request of node N_1 a direct connected relation between node N_1 and N_2 will be established. Hence, mathematically,

Direct relation of a social networks ={(N_i,N_j) : there exists and edge e_i between nodes N_i and N_j if nodes N_j takes decision to accept friend request send by node N_i for i=1,2,3……,n and j =1,2,3,…..,m and i≠j}

3.3 Establishment of Indirect relation between two nodes using decision taken from each node - (for connected set) in the Social Networks

Step1: Suppose Node N_1 send friend request to node N_2 and node N_2 accept friend request.

Steps2: When a direct relation between node N_1 and N_2 established nodes N_1 can be able to see all public friends of nodeN_2 and node N_2 can be able to see all public friends of nodes N_1.

Step3: Node N_1 can take decision to send friend request to any public friends of nodes N_2 and public friends of N_2 has an option to take decision in either accepting or rejecting friend request of node N_1.

Similarly, Node N_2 can take decision to send friend request to any public friends of nodes N_1 and public friends of N_1 has an option to take decision in either accepting or rejecting friend request of node N_2.

Step4: step 3 may be continuing by the any node of social networks.

Hence, Mathematically,

Indirect relation of a social networks ={(P_{Ni} , P_{Nj}): there exists and edge e_i between nodes P_{Ni} and P_{Nj} if nodes P_{Nj} takes decision to accept friend request send by node P_{Ni} or if nodes P_{Ni} takes decision to accept friend request send by node P_{Nj} when node (N_i,N_j) are directly connected for i=1,2,3……,n and j =1,2,3,…..,m and i≠j}.

Here, P_{Ni} and P_{Nj} denotes public nodes of nodes N_i and nodes N_j respectfully.

Decision taken to establish Mutual relation is also an indirect relation [3].

3.4 Establishment of Indirect relation between two nodes using decision taken from nodes (for disconnected set) in the Social Networks

Step1:Suppose node N_1 is a member of a social networks and have posted a public message in social networks.

Step2:There may exist different connected or disconnected nodes that can see the message of node N_1

***Case1*:** Virtual relation in social networks: Virtual relation is that relation who also considered the situation in which two nodes of social networking may or may not be connected. There exist different types of relationship between nodes of social networking like Public relationship, Private relationship, and Mutual [4]

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relationship. Virtual relationship also considered the relationship between two disconnected nodes of social networking .Virtual relation is a countable relation between the nodes of social networks .Two nodes of a social networking may or may not be virtually related. Nodes of social networks may take decision to establish a virtual relation in social networks [20].

Hence, Mathematically,

Virtual relation ={(Ni, Nj): there does not exist edges e_i between nodes N_i and N_j but nodes N_i can take decision to give their valuable remarks/comments on some specific topics to node N_j or vice-versa for i=1,2,3……,n and j =1,2,3,…..,m and i≠j}

Case2: Democratic relation in Social Networks: The word Democracy is an important term for formation of a government in a Society. The name comes from Greek for 'rule by the people'. At the most basic level, it is a type of government or political system ruled by citizens, people who are members of a society. In a democracy, citizens hold some level of power and authority, and they participate actively in the political, or decision-making, process of their government. Democracy is a form government. In the Social Networks it is observed that, nodes of social networks follow some features of democracy. Democratic relationship is essentially a relationship in a social networking, in which nodes have an opportunity to share /express/comments related to any topics [22].Hence, Mathematically,

Democratic relation ={(N_i, N_j): there may or may not exist edges e_i between nodes N_i and N_j but nodes N_i can take decision to give their valuable remark/comments on some specific topics to node N_j or vice-versa for i=1,2,3……,n and j =1,2,3,…..,m and i≠j}

4. Role of Decision Based Relation / Situation / Condition in Social Networks

4.1 Countable numbers of nodes to be related in social networks

When node N_1 decided to expand their network, it will send their friend request to N_2 ,a member of social networks. N_1 is uncertain about establishment of friend request. If N_2 accept friend request then public nodes of N_2 will be visible by nodes N_1 . So node N_1 has an option to send the friend request to public nodes of N_2 and public nodes of N_2 have an option to take decision for establishing relationship with node N_1.

Hence, in this decision based relation there exists a one-one mapping f :$N_{\text{number of connected nodes}} \rightarrow$ set of Natural number.

4.2Establishment of communication of message

Nodes of social networks can share the public message which he received by the members of social networks. Sharing of message by nodes may create a decision tree. A decision tree is a decision support tool that uses a tree-like graph or model of decisions and their possible consequences, including chance event outcomes, resource costs, and utility. In the social network, one node of social networks may take decision to share the public message of other nodes.

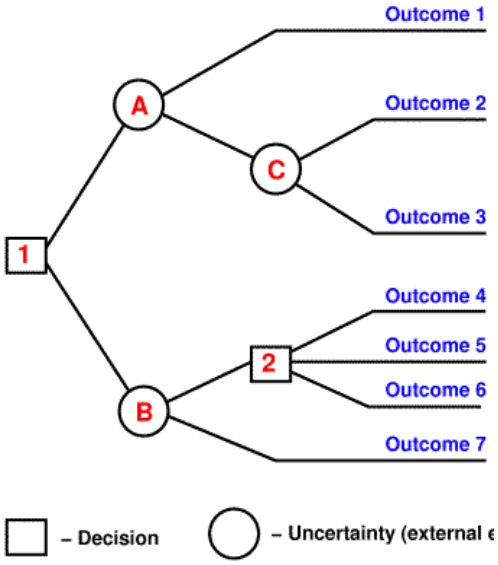

Fig.1: Diagram showingAlternative in Decision [23]

Suppose node N_1 and node N_2 are connected and node N_1 send a message to nodeN_2.Now node N_2 have option to either like/dislike/share the message. Node N_2 may take decision to do this option. In the social network it is possible to share the message to individual nodes or a group of nodes. Hence, it depends on a node to take decision to share the message individually or in a group. Therefore, sharing of message can be done into two way-

(a)Sequentially sharing of message – (Individual)-In this case message can be share one nodes to another single nodes. So this types of sharing of message may be called as Sequentially sharing of message

(b)Parallel sharing of message –(in a group)- In this case message can be share one nodes to another group nodes. So this type of sharing of message may be called as Parallel sharing of message.

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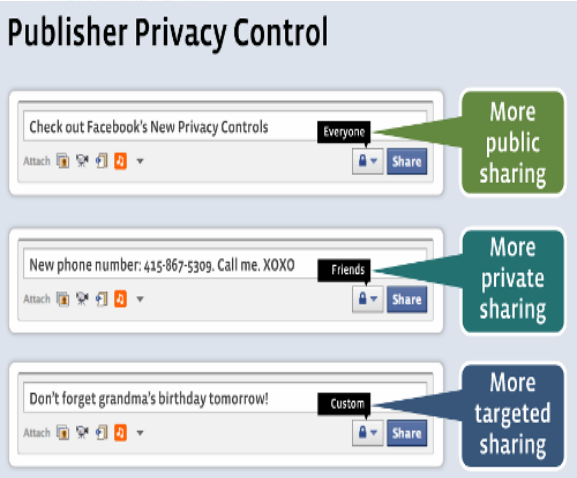

Fig.2: Diagram showing types of sharing message[25]

4.3 Maintenance of Privacy of message in social networks

A nodes of social network may receive unsolicited /unwanted messages from unknown members of social networks. A nodes of social networks may change their social networks settings to only receive messages from nodes which it know, which is the most effective way to protect against these types of messages.

But if a nodes send a message to its direct connected nodes which may be public or private. Mathematically, f: $N_1 \rightarrow N_2$ such that Message send by node N_1 is received by node N_2. Now, privacy of send message by node N_1depends on the nodeN_2 .Node N_1 will remain uncertain about privacy of this message and node N_1 has to depend on the decision of nodeN_2. Node N_2 may share or may not share this message to others members of social networks. Here, nodeN_2 is independent to either share or do not share this message which is send by node N_1 to other members of nodes.

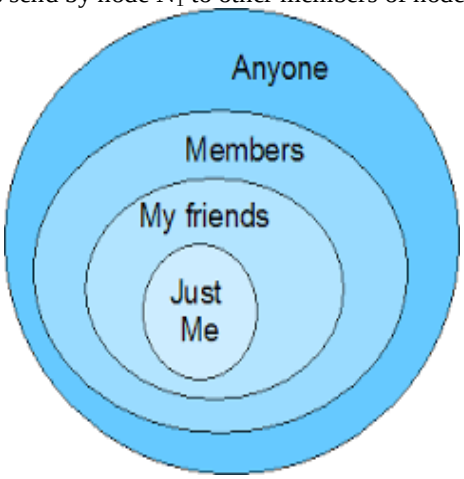

Fig.3: Diagram showing for maintaining privacy By selecting the choice of groups[26]

4.4 Decision for searching of nodes in Social networks for establishing relationship

The fundamental ideas behinds the social networks to connect the nodes. The nodes may be individual or in a group. The idea behind groups is that they are neither public nor open. They are a collection of people whom the group's admin knows, or at least has the phone number of in their contacts list. Unless the group's admin adds a nodes to the group, nodes will not be able to participate in it. Even if nodes are aware of its existence. A node of social networks may take decision to search the node in different way.

Suppose a node N_1 want to search node N_2 in a social networks for establishing a connection. Hence, node N1 have the following option to do this.

Option 1:

Node N1 can individually (one by one) search nodes N2 by the respective method of the respective social networks.

Option2:

Step1: node N_1 can search a group of social networks.
Step2:if node N_2 belongs to that group go to step 3
else go to step 1.
Step3: establish a relation between node N_1 and N_2.
Step4: stop;

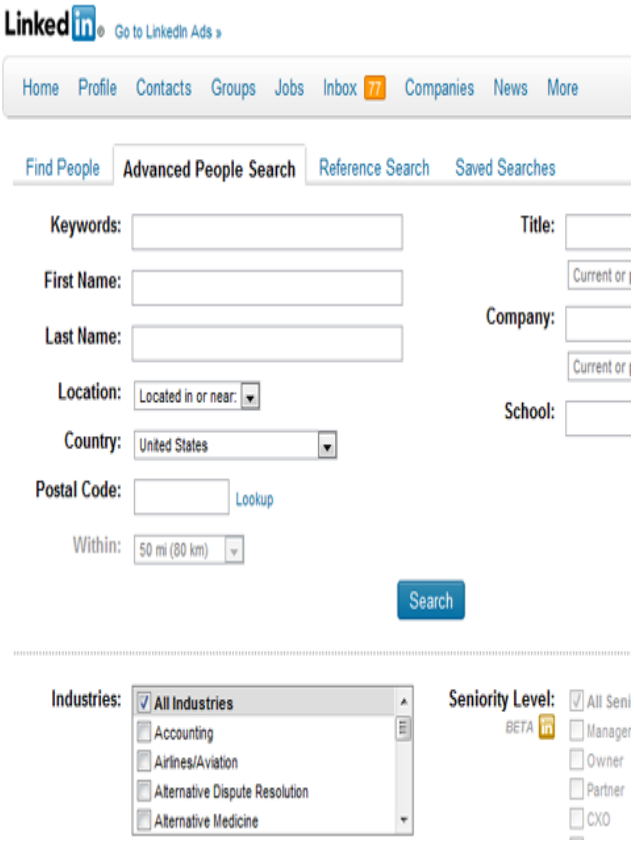

Fig.4: Diagram showing for searching of nodes in Social networks[27]

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5. CORRELATION OF COMPONENTS OF DECISION THEORY IN SOCIAL NETWORKS

Social networks mainly related with making/ creating/establishing different types of relation like direct and indirect (mutual, virtual, democratic) and communication of message among the nodes of social networks. Most of the relation in social networks is decision based. A decision maker in a social networks are a node who is responsible for making decision .The 'Act' in social networks are alternative course of action/strategies that are available for decision making and a node of social network always try to choose/do best from available alternative. After every act there will be an event in social networks. An 'Event' in social networks is results or outcomes from social networks. Events are also called 'stage of nature' or 'outcome for a decision problem'.

There are three components to any decision [28]:

(1) The choices available, or alternatives;

(2) The states of nature, which are not under the control of the decision maker;

And (3) the payoffs

5.1 Case study on connectivity of nodes

5.1.1 The choices available, or alternatives- The alternatives, or acts, are the choices available to the decision maker.

(a)In the social networks, nodes can send a friend request to any nodes of social networks.

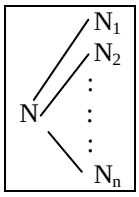

Fig.5: Friend request send by the node N to node N_1 N_2,…N_n

5.1.2 The states of nature, which are not under the control of the decision maker

In the social networks if a node sends a friend request, the choice depends on those nodes which is receiving friend request. It can either accept or reject friend request. This type of event can be correlated with characteristics function.

Characteristics function of the set:

Let us consider $A \subset E \neq \emptyset$(a universal set), then

$$f_A : E \rightarrow \{0,1\},$$

where the function ,

$$f_A(x) = 1 \text{ if } x \in A \text{ and}$$

$$f_A(x) = 0 \text{ if } x \notin A$$

is called the characters tics function of the set A.

Similarly , in social networks, let set $S \neq \emptyset$ represents an universal set and $C \subset S$, where C represented set of connected nodes, then

$$f_A : S \rightarrow \{0,1\},$$

where the function

$$f_C(x) = 1 \text{ if } x \in C \text{ and}$$

$$f_C(x) = 0 \text{ if } x \notin C$$

i.e., it is possible to represent connected nodes with symbol ' 1' and disconnected nodes with symbol '0'.

Let, node N send friend request to node N_1, N_2,….,.N_m respectively. Now it depends on the node N_1, N_2, ... ,N_m either to accept or reject friend request. Hence, state of connectivity is not the control of decision maker.

The connectivity between node N and node N_1, N_2,….,N_m can be as follows:

Connectivity	N_1	N_2	N_3	…..	N_n
N	0 or1	0or1	0or1	…….	0or1

Table I: '1' represents for acceptance of friend request and'0' represents for rejection of friend request for connectivity among node N and N_1,….N_n

Note: Here the states of nature (connection) not under the control of the decision maker node N.

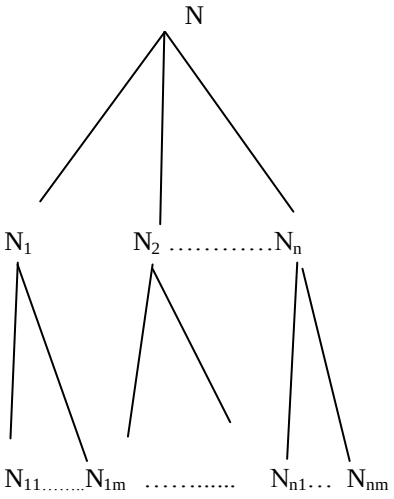

Fig.6: Diagram showing Expansion of relationship

Let, Node N_1 send friend request to node N_{11},……,N_{1m}

Node N_2 send friend request to node N_{21},……, N_{2m}.

……

……

Node N_n send friend request to node N_{n1},……,,N_{nm}

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(i) the states of nature(connection) for node N_1 and node $N_{11},N_{12},\ldots..N_{1n}$

connectivity	N_{11}	N_{12}	N_{13}	…..	N_{1m}
N_1	0 or1	0or1	0or1	…….	0or1

Table II: Connectivity among node N_1 and node $N_{11},\ldots N_{1m}$

In the above table, '1' represents for acceptance of friend request and'0' represents for rejection of friend request. Here, the states of nature (connection) not under the control of the decision maker node N_1

……
……

(ii)Similarly, the states of nature (connection) for node N_n and node $N_{n1},N_{n2},\ldots,.N_{nm}$

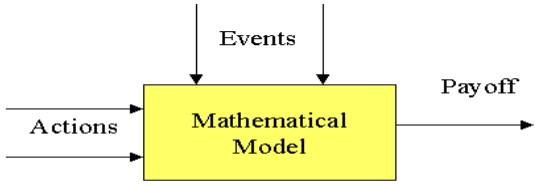
connectivity	N_{n1}	N_{n2}	N_{n3}	…..	N_{nm}
N_n	0 or1	0or1	0or1	…….	0or1

Table III: Connectivity among node N_n and node $N_{n1},\ldots N_{nm}$

In the above table, '1' represents for acceptance of friend request and'0' represents for rejection of friend request. Here, the states of nature (connection) not under the control of the decision maker node N_n

5.1.3 The payoffs

A payoff is needed to compare each combination of decision alternative and State of nature.



Components of a Probabilistic Model

Fig.7: Comonents of Probabilistic model[24]

When a node N send friend request to nodes Ni for i=1,2,3…,m; node Ni have different alternative to establish relation with node N. It can either accept or reject friend request. Also node N or Ni may establish Mutual relation or Virtual relation or democratic relationship. So, a node N and Ni may be connected in the following way in the social networks. Therefore their payoff matrix can be represented in the following way:

Node N

	Direct connection	Mutual Connection	Virtual Connection	Democratic connection	No relation and Not connected
N_1	C_{11}	C_{12}	C_{13}	C_{14}	C_{15}
N_2	C_{21}	C_{22}	C_{23}	C_{24}	C_{25}
N_3	C_{31}	C_{32}	C_{33}	C_{34}	C_{35}
: :	: :	: :	: :	: :	: :
N_m	C_{m1}	C_{m2}	C_{m3}	C_{m4}	C_{m5}

Table IV: Payoff matrix for different types of relation in Social Networks

Note: C_{ij} (for i=1,2…,m; j=1,2….5) represents cost expenditure in the establishment of relationship among respective nodes.

6. Conclusion and Future Scope

Decision making is the most important issue in all activities in social networks. There is always an uncertainty between establishment of relation between two nodes. A node has always to take decision in establishment of direct and indirect relation. Decision theory is very much applicable in social networks when nodes try to establish relationship among nodes using optimal cost. The relationship in social networks are of different types. It is the decision or choice taken by the members of social networks to develop their relationship. The decision for maintaining the relationship in social network may depend on the activity done by the members of the social networks. There is a scope for decision makers(nodes in social networks) to choose the best alternative on the basis of information available at a particular instant. There is further scope to apply the Fuzzy Decision making theory in Social Networks.

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Mr. Manoj Kumar Srivastav has been Post graduate in Pure Mathematics from University of Calcutta in year 2004 with Special paper in advanced functional analysis and category theory, universal algebra and lattice theory. At present he is doing MCA from IGNOU at St.Xavier's college as study center and he is working as a postgraduate teacher in Mathematics in an esteemed institution. He has more than 14 published research papers in National and International Journals.

Dr. Asoke Nath is Associate Professor in the Department of Computer Science. At present he is doing research work in Cryptography and Network Security, Steganography, Green Computing, e-learning, Mathematical formulation of computer Language, Mathematical modeling of social networks etc. He has published more than 158 research papers in National and International Journals and conference proceedings.

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